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Unit-Linked Life Insurances as a Form of Protection for the Future

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Abstract: Insurances with capital funds (unit-linked, UFK), i.e. life and annuity saving and covering insurances combined with shares in selected investment funds of financial institutions are considered as additional form of pension saving. However, they are frequently seen as uneconomical products, which is caused mainly by the fact that insurance policies compensations does not live up to expectations of the insured. It became an impulse to implement the analysis of cash flow characteristic of such insurance types, carry out an accurate valuation of the unit-linked insurance portfolio, as well as to investigate financial surplus in relation to a traditional insurance with guaranteed sum.

Keywords: unit-linked insurance, European option, valuation, value of insurance, Monte-Carlo method

JEL codes: C58, G22, G17

1. Introduction

Insurances related to the financial market can be considered an extension of traditional life insurances. Such contracts, so called unit-linked insurances (UFK), are complex insurance products combined with investment funds. In other words, they are life and annuity saving insurances integrated with shares in selected investment funds of financial institutions. Nevertheless, they are frequently thought of as uneconomical products, which is caused mainly by the fact that the policy compensation does not fulfil expectations of the insured. It should be pointed out that in unit-linked insurances, in contrast to traditional life insurances, the sum of insured is not usually guaranteed by the insurer and depends on the marketable securities value in which funds are deposited, therefore is a random variable. Next, it is important to realize that insurances with capital fund

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bring many investment possibilities, but in each case there is a financial risk determining the unit-linked policies value. This issue has been addressed in literature, taking into consideration formal, legal, actuarial and financial aspects, mainly from the perspective of the insurer. In Poland insurances with capital fund are insurances in which the risk is taken by the insured, who is consequently responsible for any negative outcomes of their decisions. Thus it has become a premise to analyses these insurance types from the point of view of the insured and their benefits. For this reason valuation of cash flow of the selected unit-linked products was implemented, and on this basis the value of reference portfolio was determined, account to be taken of both insurance and financial risks. Moreover, disbursements from entering a life insurance with capital fund agreement was compared to those provided by traditional life insurances with fixed net premiums.

2. Life insurance with capital fund

Financial institutions having unit-linked on offer are insurance companies. However, this insurances are not separate products; they supplement basic products, i.e. covering life insurances.

The idea of unit-linked insurances is determined by their complex savings-investment nature. In short, they allow the insured to invest a fraction of the premium into funds of various risk levels offered by insurance companies. In this way they flexibly combine the aspects of insuring and saving, at the same time creating many opportunities of managing the saved capital. In Poland such agreements enable the insured to collect savings in an individual investment portfolio, comprising various investment funds managed by external fund insurance companies independent from the insurer. What is more, the structure of unit-linked policies is open and transparent, which gives the insured the possibility of adjusting the content of the portfolio according to changing market conditions. Consequently, profits gained by these funds can vary. Unit-linked insurance are available predominantly within the scope of individual life insurances in the following forms (Moller T.,Steffensen M. 2007):

- Unlimited life insurance.
- Limited life and annuity insurance.
- Saving insurance.

Regardless of the form, the insurance with capital fund is an agreement concerning life and annuity concluded between the insured and the insurer, according to which the insured pays premiums and

in turn the insurance company provides the insured with certain payments. In contrast to traditional life insurances, in which the insurance cost (expressed in the paid premium) is constant during the entire insurance period and is not a consequence of the risk level, but of the average risk during the entire insurance period, unit-linked insurances take into account the age of the insured, so the cost is subject to payments, interest rate, administrative costs, and the risk of death, accidents, etc. As far as payments are concerned, traditional insurances stipulate their amount in the agreement as the insurance sum, whereas unit-linked insurances determine the amount of provisions basing on the value of the investment portfolio, as well as on trends concerning the price of a given fund, which are dependent on insurance policies. Due to the investment strategy, the insurance capital funds differ in terms of risk, and thus one can distinguish balanced, debt, cash and shares funds.

3. Investment portfolio and its value in UFK insurances

In unit-linked insurances like in traditional insurances, the insurance company obliges itself to the disbursement of payments according to the type of contract:

- Due to the insurance termination (while the insured is still alive) (UD).
- In case of death during the insurance period (UZ).

Contrary to traditional life insurances, in unit-linked insurances the moment of disbursement, as well as the amount of the payment, are random. At the moment of an event covered by the insurance the market value of the insurance portfolio is paid out by the insurer. Therefore the payment amount depends either on an index development or on a specific portfolio value. In relation to such construction of unit-linked the payment is a function of a cumulated investment, i.e. the value of insurance portfolio (Bacinello 2003):

$$B_t = f(FV_t)$$

where: FV_t – value of the insurance (reference) portfolio at the moment of *t.*

In practice most frequently the function *f* takes one of the following forms (Bacinello 2003):

$$f(FV_t) = FV_t \quad \text{lub} \quad f(FV_t) = \max\{G_{\Pi}, FV_t\},$$

where: G_{Π} – guaranteed sum dependent on the amount of the insurance premium.

In the former case the disbursement amount depends only on the value of the reference portfolio, whereas in the latter the minimal insurance sum is guaranteed by the insurer. The second instance refers to a situation in which at the moment of an event covered by the insurance the higher value

is paid out, i.e. the minimal guaranteed insurance sum with added marked value of the insurance portfolio. The majority of Polish insurance companies tend to minimize the covering aspect of insurance, at the same time focusing on the saving feature. Consequently, agreements of this type do not imply any assurance, which leads to implementing the first presented formula while paying out the insurance amount. The value of the insurance portfolio in such insurances $X(t)$ is random and depends on:

- The fund price at the moment of t.
- The past units price (covered by the premiums).
- The investment size.

Thus the insurance portfolio value is achieved by investing a fraction of the paid insurance premium. The fraction of the premium invested at the moment t_i is marked as Π_i and referred to as the investment premium. Accordingly, the insured builds their reference portfolio value by investing in chosen assets (either investment funds or shares), with price determined as the process $S(t)$. The value balance at the moment t_i (i.e. at the end of the year) is described by the following formula (Homa 2013):

$$FV_{t_i} = \underbrace{FV_{t_{i-1}}}_{\text{portfolio value in the moment } t_{i-1}} + \underbrace{FV_{t_{i-1}} \cdot \frac{S_{t_i} - S_{t_{i-1}}}{S_{t_{i-1}}}}_{\text{gains from the investment in the year } i} + \underbrace{\Pi_i}_{\text{changes on the account in the year } i}$$

Hence the portfolio value is the sum of portfolio values in the period before achieving the increased value due to the profits from investments and changes on the account. At each moment t_i the insured purchases respectively $\Pi_i \cdot (S_{t_i})^{-1}$ asset units for a price Π_i. So at the moment t_i the portfolio comprises $\sum_{i=1}^{j-1} S_{t_i}^{-1} \cdot \Pi_i$ units, where each of them possesses the value S_{t_i}. Therefore the reference portfolio value at the moment of the policy termination equals:

$$FV_T = \sum_{i=1}^{n-1} \frac{S_T}{S_{t_i}} \Pi_i$$

which means that the discounted portfolio value (in relation to S) equals the discounted value of all investments. Thus the reference portfolio value at any time can be described by the following formula:

$$FV_t = \sum_{i=0}^{\min\{i|t_i>t\}-1} \frac{S_t}{S(t_i)} \cdot \Pi_i$$

Taking into account the money value changing over time, the market value of the disbursed payment at the moment t equals:

$$V(B_T,t)=\frac{\upsilon(T)}{\upsilon(t)}\sum_{i=0}^{n-1}\frac{S_T}{S_{t_i}}\Pi_i$$

Making the valuation one should take into consideration not only the market value, but also the money value changing over time and the risk related to the subject of insurance. As far as life and annuity insurances are concerned, it would be also the mortality risk. If these two types of risk are considered to be independent, the valuation at the end of the insurance period would be implemented by stipulating the actuarial value in the following way:

$$E[V(B_T,t)]=\frac{\upsilon(T)}{\upsilon(t_0)}E\left[\sum_{i=0}^{n-1}\frac{S_T}{S_{t_i}}\Pi_i \mid \mathrm{I}_{[K_x>T]}\right]={}_{T-t}p_{x+t}\,\frac{\upsilon(T)}{\upsilon(t_0)}E\left[\sum_{i=0}^{n-1}\frac{S_T}{S_{t_i}}\Pi_i\right]$$

In case of disbursement valuation at the moment of death of the insured at any time during the insurance period, i.e. in the period $[t_{j-1},t_j)$, the probability of death ${}_{t_{j-1}}p_{x+t}\cdot{}_{t_j-t_{j-1}}q_{x+t_{j-1}}$ is taken into account. Bearing in mind all of the above, it can be concluded that a person who purchases a unit-linked policy can expect a surplus to the guaranteed insurance sum in the traditional life insurance only in the situation when the insurance portfolio value exceeds the guaranteed value. Due to the fact that the unit-linked insurances structure implies financial risk, the insured should control the risk included in the fraction of the premium which constitutes the investment portfolio value. The higher the premium, the greater the investment portfolio value, and the insured receives a compensation resulting from the surplus coming from the investment portfolio value.

4. Financial and mortality market model

As an example both traditional and unit-linked terminated agreements concerning life and annuity insurance were examined, with the premise that in the case of death at the moment $\tau_i\epsilon\Theta/\{\tau_0\}$ the insurer will disburse a sum resulting from the portfolio value:

$$\frac{\upsilon(\tau_{i+1})}{\upsilon(t_0)}k\Pi\sum_{j=0}\frac{S(\tau_{i+1})}{S(t_j)}$$

Analogical payment would be issued also in the situation of the insured being alive at the moment of the contract termination. Making the valuation of both traditional and unit-linked insurances, one has to take into account not only the money value change over time, but also an accurate description of mortality dynamics. It has been therefore accepted that on the market there is an account with fixed interest rate and continuous capitalization, which means that the monetary unit value process equals (Weron 1998):

$$B_t = e^{\delta t} = e^{t \ln(1+r)}$$

Furthermore, in order to create an accurate description of the mortality rate in the population sample aged from 30 to 80, the Gompertz-Makeham law was implemented, which allowed to obtain the probability of death. The rate of deceases can be described by the formula (Bowers 1997):

$$\mu(x+t) = A + Bc^{x+t}$$

Basing on Life Table for men, a function approximation was carried out, which led to the following estimators of the biggest plausibility of the function's parameters:

$$A = 0{,}0004;\ B = 0{,}0000034674;\ c = 10^{0{,}06}$$

Therefore it was possible to obtain the probability of both being alive and dead used in the example.

Additionally, in unit-linked insurances a fraction of the paid insurance premium (so called saving premium) is invested by the insurance company into the financial market. Thus the effectiveness of insurance companies is increased, and their market offer becomes more competitive. Nevertheless, since insurance companies do not know in advance consequences of such an activity, the risk seems to be taken by the insured. In this case the life insurance cash value is influenced by both positive and negative effects of this activity. Also, it was assumed that the insured can differentiate profits resulting from the variable portfolio value by implementing various investment strategies. In the analysis the best UFK funds in the following four portfolio groups were taken into account:

- AK_UN – general shares,
- MI_ST – mixed of balanced growth,
- MI_ZR – mixed balanced,
- PD_UN – general of debt

The analysis of funds risk was conveyed in these groups with regard to basic measures: standard deviation, Sharp's index, and Information Ratio (IR). The fund with the highest note was chosen from each group; outcomes are presented in the table 1.

Table 1. Indices of the best funds in groups (1) – (4)

Group	The best fund in a group	S	Sharpe's Index	IR	Declared Benchmark
AK_UN	AXA Quercus Aggressive UFK	4.29	0.09	0.45	100% WIG
MI_ZR	Nordea Balanced Portfolio	2.35	0.08	0.53	50% WIG + 50% Polish Treasury Bonds Market Index
MI_ST	Nordea Balanced Growth Portfolio	1.60	0.06	0.57	20% WIG + 80% Polish Treasury Bonds Market Index
PD_UN	HDI Generali Novo Debt Securities	0.86	0.23	0.4	100% EFFAS Bond Indices Poland Liquid All > 1Yr,

Source: Own elaboration based on the data from www.analizy.pl.

What follows, in order to carry out the valuation of the unit-linked flow, as well as to calculate the amount of payment, a simulation of the prices process has to be implemented (with attention paid to the presumed financial market model). The Black-Scholes (Black-Merton-Scholes) model with horizon T has been adopted as the financial market model. It has been assumed that an ideal market is being dealt with, where there are units of the fund chosen by the insured with the price estimated by the formula (Milevsky, Salisbury 2006):

$$dS_t = \mu_S(t)S_t dt + \sigma_S(t)S_t dW_t$$

where W_t is the Wiener process (the geometrical Brownian motion), S_t – the presumed price of the derivative contract, S_0 – the actual price of the derivative contract, μ- the process average, σ – standard deviation of the process. The only solution of the above differential equation is following process which has lognormal distribution:

$$S_t = S_0 \exp\left(\sigma W_t + \left(\mu - \tfrac{1}{2}\sigma^2\right)\cdot t\right)$$

So the simplest model of price evolution, the geometrical Brownian motion, has been accounted for. Upon application of this mathematical model the following formula MC simulating (Glasserman 2004) the future value of the derivative contract can be created:

$$S^i_{t_k} = S^i_{t_{k-1}} \exp\left[\left(r - \frac{\sigma^2}{2}\right)\left(t_k - t_{k-1}\right) + \sigma\sqrt{t_k - t_{k-1}}\,\varepsilon^i_k\right]$$

where ε^i_k means independent values generated from the normal distribution, r is the risk-free interest rate.

5. Simulation of portfolio and payment value for an exemplary unit-linked insurance in relation to investment strategy

Regardless of the form, i.e. traditional or unit-linked life insurance should be a long-term solution. Therefore in the valuation of the capital increasing effectiveness within the unit-linked system a long time horizon has been applied, according to which the unit-linked payment amount (i.e. the reference portfolio value) and a traditional life and annuity insurance sum were compared. It was accepted that the insured is a man aged x who has purchased an n-year life and annuity insurance policy with the insurance sum $G_{\Pi} = 1000$ monetary unit. Next, the unit-linked actuarial value of reference portfolio was calculated by applying the Monte Carlo method, with various investment strategies implemented. The capitalisation complied to the agreed scenario: 10^8 iterations, for 20-, 30-, 40-, and 50-year long unit-linked contracts (Table 2).

Table 2. Actuarial value of reference portfolio with different strategies at the moment n

Portfolio	n=20	n=30	n=40	n=50
AK_UN	1501,49	1511,51	1506,64	1513,63
MI_ZR	1498,28	1502,22	1500,08	1497,55
MI_ST	1500,41	1497,94	1485,26	1498,05
PD_UN	1476,00	1393,17	1407,43	1489,56

Source: Own elaboration.

Taking into consideration the above data concerning the actuarial value of the unit-linked life insurance portfolio at time n, it can be presumed that both the duration of the insurance, as well as its investment strategy applied by the insured, do not significantly determine the policy value. More importantly, the expected value in each of the presented cases exceeds the traditional insurance sum, which means that regardless of the chosen investment strategy, the insured can anticipate a surplus in comparison to traditional life insurances. Contrary to traditional insurances it is not a sufficient evidence, because the portfolio value is a random variable, which can take different value during the insurance period. Therefore moment of payment is also important and we examined the actuarial value of the payment during the period of insurance for n=20 and n=50 (Table 3 and 4).

Table 3. Actuarial value of reference portfolio with different strategies and $0 \leq t \leq 2$ $0= n$

n=20	AK_UN	MI_ZR	MI_ST	PD_UN
t=5	709,1	708,86	709,02	697,99
t=10	912,4	910,14	912,12	898,87
t=15	1170,75	1164,67	1170,27	1168,06
t=20	1501,49	1498,28	1500,41	1476

Source: Own elaboration.

Table 4. Actuarial value of reference portfolio with different strategies and $0 \leq t \leq 5$ $0= n$

n=50	AK_UN	MI_ZR	MI_ST	PD_UN
t=5	259,3	203,41	203,65	203,19
t=10	336,71	390,04	334,99	260,71
t=15	432,62	473,03	413,41	334,46
t=20	556,73	549,69	551,93	475,17
t=25	708,29	640,48	706,02	709,51
t=30	955,22	913,84	908,35	908,03
t=35	1167,29	1169,35	1170,79	1011,68
t=40	1289,57	1266,25	1287,18	1114,03
t=45	1401,13	1359,02	1353,72	1355,27
t=50	1513,63	1497,55	1498,05	1489,56

Source: Own elaboration.

Table 5. Actuarial value of surplus with different strategies and $0 \leq t \leq 5$ $0= n$

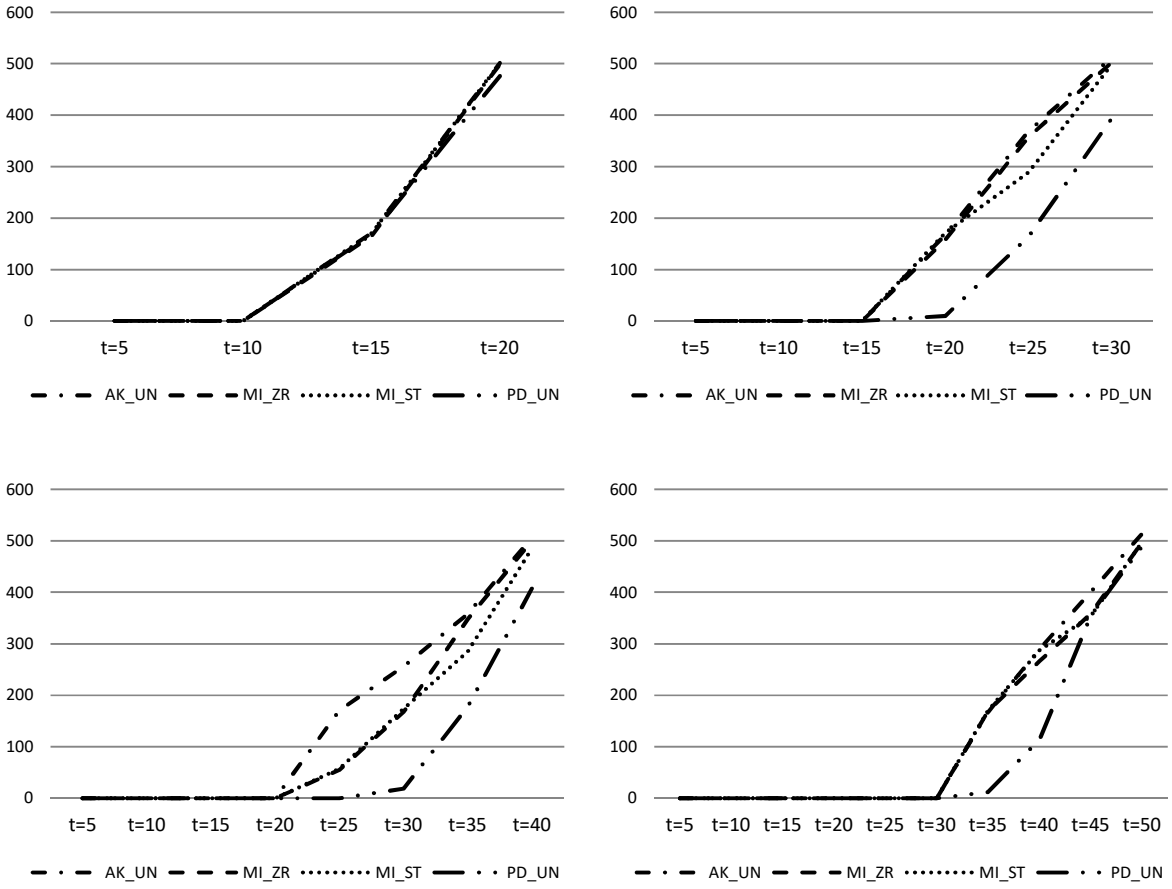
investment portfolio		AK_UN	MI_ZR	MI_ST	PD_UN
actuarial payment	t=5	-290,9	-291,14	-290,98	-302,01
error		0,41	0,35	0,24	0,09
actuarial payment	t=10	-87,6	-89,86	-87,88	-101,13
error		0,79	0,67	0,45	0,17
actuarial payment	t=15	170,75	164,67	170,27	168,06
error		1,31	1,13	0,74	0,28
actuarial payment	t=20	501,49	498,28	500,41	476
error		2,09	1,8	1,11	0,42

Source: Own elaboration.

Comparing unit-linked insurance and classic life insurance analyzed also surplus values with its probability during the insurance period. Outcomes of the simulation, implemented with the MC method for a 20-year long UFK agreement with capitalisation compatible with the agreed scenario for different strategies along with the estimation error, are presented in Table 5.

Analogical simulations were carried out for unit-linked with the insurance period $n = 30, 40$ and 50, respectively, and the outcomes are presented in the following figures.

Figure 1. Surplus unit-linked insurance vs classic life insurance during the insurance period with n = 20, 30, 40, 50



Source: own elaboration.

The obtained outcomes indicate explicitly that the unit-linked insurance reference portfolio value exceeds the insurance sum only after certain time within the agreement duration, which means that only then the insured can expect a financial surplus resulting from unit-linked. Also, the long-term feature has been confirmed. Therefore an insured person resigning from the unit-linked policy too early, regardless of the agreed investment strategy, is not only disadvantaged in comparison to traditional insurances, but also does not get the paid premiums back.

6. Conclusion

Having considered the obtained outcomes it can be stated that regardless of the agreed way of capitalising, the insured can control the surplus resulting from the UFK insurance portfolio value compared to traditional insurance. The amount of the UFK insurance disbursement at the moment of policy termination exceeded the insurance sum guaranteed by traditional insurances independently of the risk-aversion of the insured and the agreed investment strategy. Nevertheless, it should be pointed out that the surplus over the guaranteed sum is present in the middle of the insurance period, and its dynamics is influenced by the investment strategy. Therefore, taking into account the long-term character of UFK, the combination of covering and saving is a favourable solution. Finally, while choosing UFK one has to bear in mind that these are long-term products, and in this framework they can be considered an effective way of collecting pension savings.

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MAGDALENA HOMA

Ubezpieczenia na życie z funduszem kapitałowym jako forma zabezpieczenia na przyszłość

Streszczenie

Ubezpieczenia z funduszem kapitałowym UFK czyli oszczędnościowo-ochronne ubezpieczenie na życie i dożycie połączone z udziałami w wyselekcjonowanych funduszach inwestycyjnych instytucji finansowych wskazywane są jako dodatkowa forma oszczędzania na emeryturę. Coraz częściej jednak pojawiają się opinie wskazujące na nieopłacalność tego typu produktów, a wynikają one przede wszystkim z faktu, że wypłata z tytułu zawartej polisy nie spełnia oczekiwań ubezpieczonych. Stanowiło to przesłankę do przeprowadzenia analizy przepływów pieniężnych charakterystycznych dla tego typu ubezpieczeń oraz dokonanie prawidłowej wyceny wartości portfela ubezpieczeniowego UFK oraz zbadane nadwyżki finansowej w stosunku do klasycznego ubezpieczenia z gwarantowaną sumą.

***Słowa kluczowe**:* ubezpieczenie unit-linked, opcja europejska, wycena, wartość ubezpieczenia, metoda Monte-Carlo.